Using the Standard Solar Model to Constrain Composition and S-Factors

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While standard solar model (SSM) predictions depend on approximately 20 input parameters, SSM neutrino flux predictions are strongly correlated with a single model output parameter, the core temperature T_c. Consequently, one can extract physics from solar neutrino flux measurements while minimizing the consequences of SSM uncertainties, by studying flux ratios with appropriate power-law weightings tuned to cancel this T_c dependence. We re-examine an idea for constraining the primordial C+N content of the solar core from a ratio of CN-cycle ^{15}O to pp-chain ^{8}B neutrino fluxes, showing that nonnuclear SSM uncertainties in the ratio are small and effectively governed by a single parameter, the diffusion coefficient. We point out that measurements of both CN-I cycle neutrino branches – ^{15}O and ^{13}N β-decay – could in principle lead to separate determinations of the core C and N abundances, due to out-of-equilibrium CN-cycle burning in the cooler outer layers of the solar core. Finally, we show that the strategy of constructing "minimum uncertainty" neutrino flux ratios can also test other properties of the SSM. In particular, we demonstrate that a weighted ratio of ^{7}Be and ^{8}B fluxes constrains a product of S-factors to the same precision currently possible with laboratory data.

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Important developments in solar neutrino physics have occurred over the past one to two years that impact the field's two major goals, probing the core of the Sun and constraining new weak-interaction phenomena. Super-Kamiokande IV has reported a ^{8}B ν flux measurement, $(2.34 \pm 0.03 \pm 0.04) \times 10^6/\mathrm{cm}^2\mathrm{s}$ [1], that continues the progress toward high precision: combining in quadrature the statistical and systematic errors, one finds that the Super-Kamiokande uncertainty is now about 2%. The SNO combined analysis of solar neutrino data from all phases, has significantly narrowed the allowed range for the mixing angle θ_{12} [2], and new reactor and accelerator neutrino results have fixed the contributions of the subdominant mixing angle θ_{13} [3]. A new round of standard solar model (SSM) calculations has been completed to explore competing compositions that optimize SSM agreement either with the solar interior properties (as determined from helioseismic mappings of the sound speed) or with solar surface properties (the interpretation of photoabsorption lines using our most sophisticated model of the Sun's atmosphere) [4]. The nuclear physics of the SSM has also been updated, with completion of the nuclear astrophysics community's second decadal evaluation of SSM S-factors and weak interaction rates [5]. Finally, Borexino has produced a precise (4.5%) measurement of the 862 keV neutrinos from ^{7}Be decay and first results on the pep flux, following its successful calibration campaign [6]. The ^{7}Be result has sharpened the "luminosity constraint" on the pp/pep neutrinos, which currently provides our most precise constraint on these fluxes if one assumes a steady-state Sun.

The SSM, despite its relatively simple underlying physics, depends on ~ 20 input parameters, including the solar age and luminosity, the opacity, the rate of diffusion, the zero-age abundances of key elements (He, C, N, O, Ne, Mg, Si, S, Ar, Fe), and the S-factors for the pp chain (responsible for 99% of solar energy generation) and CN cycle. While SSM neutrino flux predictions generally change with variation in any of these input parameters, it has been recognized for some time that flux predictions are strongly correlated with a single output parameter, the core temperature T_c [7]. That is, a multi-dimensional set of variations in SSM input parameters $\{\Delta\beta_j\}$ from the SSM best values $\{\beta_j^{\mathrm{SSM}}\}$ often collapses to a one-dimensional dependence on T_c, where T_c is an implicit function of the variations $\{\Delta\beta_j\}$: the effect of any variation $\{\Delta\beta_j\}$ on ϕ_i can be estimated simply from its effects on T_c. The dominance of T_c as the controlling parameter for neutrino fluxes reflects the sensitivity of Maxwellian-averaged rates $\langle v\sigma(E)\rangle$ to temperature. Consequently, if $\phi_{i_1} \propto T_c^{x_{i_1}}$ and $\phi_{i_2} \propto T_c^{x_{i_2}}$, one can form weighted ratios $\phi_{i_1}/\phi_{i_2}^{x_{i_1}/x_{i_2}}$ that are nearly independent of T_c and thus nearly independent of variations over the multi-dimensional parameter space $\{\Delta\beta_j\}$.

The situation becomes more interesting when two fluxes, say ϕ_{i_1} and ϕ_{i_2}, in addition to their common dependence on most underlying parameters $\{\beta_j\}$, have a very different dependence on some specific parameter, say β_1. In that case, from a weighted ratio of observed fluxes, one might be able to learn something about β_1, while the dependence on other input parameters largely cancels out. An example worked out previously [8] and

updated below, is the additional linear dependence of CN neutrino fluxes on the primordial core number densities of C and N. To the extent that SSM uncertainties exceed the uncertainty of neutrino flux measurements, new information can be extracted from neutrino measurements – in this case, the abundances of C and N in the Sun's primordial core. Another example we will discuss is the possibility that solar neutrino fluxes can be used to cross-check laboratory measurements of S-factors.

The correlations between ϕ_i and T_c are strong but not exact: for example, as the neutrino-producing core is extended (with the extent depending on the neutrino source), fluxes must depend on an integral over a core temperature profile, which cannot be exactly proportional to T_c for all variations. Modern SSM calculations allow one to address such issues, and to include their effects in the analysis. Monte Carlo studies can be done over wide classes of parameter variations $\{\Delta\beta_j\}$, determining not only the best power-law descriptions of the fluxes, but to also the extent of reasonable variations around the power-law estimate.

The sensitivity to parameter variations can be be expressed in terms of the logarithmic partial derivatives $\alpha(i,j)$ evaluated for each neutrino flux ϕ_i and each SSM input parameter β_j,

$$\alpha(i,j) \equiv \frac{\partial \ln \left[\phi_i/\phi_i^{\rm SSM}\right]}{\partial \ln \left[\beta_j/\beta_j^{\rm SSM}\right]} \tag{1}$$

where $\phi_i^{\rm SSM}$ and $\beta_j^{\rm SSM}$ denote the SSM best values. This information, in combination with the assigned uncertainties in the β_j, then provides an estimate of the uncertainty in the SSM prediction of ϕ_i. Here we employ the logarithmic partial derivatives of [4], which were evaluated for two different metallicities, corresponding to the higher Z composition of [9], denoted GS98, and the lower Z composition of [10], denoted AGSS09. The older GS98 abundances were obtained from a simple analysis of the solar atmosphere and yield excellent agreement with interior helioseismology. The newer AGSS09 abundances, obtained from a more sophisticated 3D model of the solar atmosphere that significantly improves the agreement between measured and observed lines, are $\sim 30\%$ lower, and produce SSM sound speed profiles in significant disagreement with helioseismology. The SSMs evolved from these compositions are denoted SFII-GS98 and SFII-AGSS09 in this paper, where SFII (Solar Fusion II) indicates the use of the latest nuclear S-factors [5].

The logarithmic partial derivatives for the SFII-GS98 SSM are given in Tables I and II, divided as in [8] into two sets, corresponding to nuclear and "environmental" β_js. The nuclear parameters are the S-factors for the pp chain and CN cycle: S_{11} (p+p β decay), S_{33} (^{3}He(^{3}He,pp)^{4}He), S_{34} (^{3}He(^{4}He,γ)^{7}Be), S_{17} (^{7}Be(p,γ)^{8}B), S_{e7} (^{7}Be electron capture), and S_{114} (^{14}N(p,γ)^{15}O). The "environmental" parameters are those that directly influence the local temperature in the Sun, e.g., through their effects on evolution, the opacity, or SSM boundary conditions. They include the luminosity $L_\odot$, the Sun's age, the diffusion parameter, and the opacity. They also include the mass fractions of the principal solar metals, C, N, O, Ne, Mg, Si, S, Ar, and Fe, which have a significant influence on the opacity due to the strength of free $\leftrightarrow$ bound transitions.

The partial derivatives allow one to define the power-law dependencies of neutrino fluxes, relative to the SSM best-value prediction $\phi_i^{\rm SSM}$

$$\frac{\phi_i}{\phi_i^{\rm SSM}} = \prod_{j=1}^{N} x_j^{\alpha(i,j)} \quad \text{where} \quad x_j \equiv \frac{\beta_j}{\beta_j^{\rm SSM}} \tag{2}$$

and where the product extends over 19 SSM input parameters of Tables I and II. These derivatives determine how SSM flux predictions will vary, relative to $\phi_i^{\rm SSM}$, as the β_j are varied from their SSM best values.

Our first example of the use of the logarithmic partial derivatives follows [8], though the analysis here differs in certain respects. The Sun produces about 1% of its energy through the CN-I cycle, which produces νs from the reactions ^{13}N(β^+)^{13}C and ^{15}O(β^+)^{15}N, with respective β-decay endpoints of 1.20 MeV and 1.73 MeV. Their fluxes in the SFII-GS98 SSM are

$$\begin{aligned} \phi(^{13}\mathrm{N}) &= 2.96(1 \pm 0.14) \cdot 10^8/\mathrm{cm}^2\mathrm{s} \\ \phi(^{15}\mathrm{O}) &= 2.23(1 \pm 0.15) \cdot 10^8/\mathrm{cm}^2\mathrm{s}. \end{aligned} \tag{3}$$

The primordial C and N in the solar core are the catalysts for the conversion of four protons to ^{4}He via the CN cycle: the CN cycle alters the ratio of C to N as its burns into equilibrium, but does not change the total number density of C+N. The additional linear dependence of the CN-cycle on metallicity, due to this dependence on primordial C and N, can be isolated by forming a ratio of fluxes that is effectively independent of T_c, under variations in all other SSM parameters. The appropriate ratio can be identified either by SSM Monte Carlo studies, at considerable cost numerically, or estimated from the logarithmic partial derivatives. It was shown in [8] that the two approaches yield essentially the same answer.

The neutrino flux ratio identified in this way has the requisite metal sensitivity to distinguish GS98 abundances from those of AGSS09, resolving the solar abundance problem. More fundamentally, it will allow us to make a important test of a key SSM assumption, that the primordial Sun was homogeneous when nuclear burning began – an assumption not obviously correct, given what we have learned in the past decade about large-scale metal segregation in the protoplanetary disk [8], but nevertheless critical to the SSM, which uses solar surface abundances to fix core abundances in the primordial Sun. Several groups have discussed relaxation of this assumption as a possibility for reconciling helioseismic data with AGSS09 abundances [8, 11–13].

TABLE I: Partial derivatives $\alpha(i,j)$ of neutrino fluxes with respect to solar environmental parameters and S-factors. Table entries are the logarithmic partial derivatives $\alpha(i,j)$ of the solar neutrino fluxes ϕ_i with respect to the indicated solar model parameter β_j, taken about the SFII-GS98 SSM best values [4]. Several flux ratios that reduce the solar environmental factors are shown.

	Environmental β_j				Nuclear β_j					
Source	$L_\odot$	Opacity	Age	Diffusion	S_{11}	S_{33}	S_{34}	S_{17}	S_{e7}	S_{114}
ϕ(pp)	0.766	-0.112	-0.100	-0.013	0.105	0.034	-0.067	0.000	0.000	-0.007
ϕ(pep)	0.989	-0.318	-0.024	-0.019	-0.217	0.049	-0.097	0.000	0.000	-0.010
$\phi(^7\mathrm{Be})$	3.434	1.210	0.760	0.126	-1.024	-0.428	0.853	0.000	0.000	-0.001
$\phi(^8\mathrm{B})$	6.914	2.611	1.345	0.267	-2.651	-0.405	0.806	1.000	-1.000	0.007
$\phi(^{13}\mathrm{N})$	4.535	1.487	0.932	0.337	-2.166	0.031	-0.062	0.000	0.000	0.747
$\phi(^{15}\mathrm{O})$	5.942	2.034	1.364	0.382	-2.912	0.024	-0.052	0.000	0.000	1.000
$\phi(^7\mathrm{Be})/\phi(^8\mathrm{B})^{0.465}$	0.219	0.002	0.135	-0.004	0.209	-0.240	0.478	-0.465	0.465	-0.004
$\phi(^{13}\mathrm{N})/\phi(^8\mathrm{B})^{0.576}$	0.553	-0.017	0.157	0.183	-0.639	0.264	-0.526	-0.576	0.576	0.743
$\phi(^{15}\mathrm{O})/\phi(^8\mathrm{B})^{0.785}$	0.515	-0.016	0.308	0.172	-0.831	0.342	-0.685	-0.785	0.785	0.995
$\phi(^{13}\mathrm{N})/\phi(^{15}\mathrm{O})^{0.776}$	-0.075	-0.091	-0.126	0.041	0.093	0.012	-0.022	0.000	0.000	-0.029

TABLE II: As in Table I, but for the partial derivatives $\alpha(i,j)$ with respect to the fractional abundances of the primordial heavy elements. .

	C, N β_j		Environment Abundance β_j						
Source	C	N	O	Ne	Mg	Si	S	Ar	Fe
ϕ(pp)	-0.008	-0.002	-0.006	-0.005	-0.004	-0.010	-0.007	-0.002	-0.021
ϕ(pep)	-0.016	-0.003	-0.012	-0.006	-0.003	-0.013	-0.015	-0.005	-0.062
$\phi(^7\mathrm{Be})$	0.002	0.001	0.062	0.055	0.050	0.104	0.076	0.019	0.207
$\phi(^8\mathrm{B})$	0.027	0.007	0.139	0.109	0.092	0.192	0.140	0.035	0.502
$\phi(^{13}\mathrm{N})$	0.856	0.165	0.082	0.058	0.049	0.111	0.081	0.021	0.294
$\phi(^{15}\mathrm{O})$	0.815	0.217	0.112	0.081	0.069	0.150	0.109	0.028	0.397
$\phi(^7\mathrm{Be})/\phi(^8\mathrm{B})^{0.465}$	-0.011	-0.002	-0.003	0.004	0.007	0.015	0.011	0.003	-0.026
$\phi(^{13}\mathrm{N})/\phi(^8\mathrm{B})^{0.582}$	0.840	0.161	0.002	-0.005	-0.004	0.000	0.000	0.001	0.005
$\phi(^{15}\mathrm{O})/\phi(^8\mathrm{B})^{0.785}$	0.794	0.212	0.003	-0.005	-0.003	-0.001	-0.001	0.001	0.003
$\phi(^{13}\mathrm{N})/\phi(^{15}\mathrm{O})^{0.776}$	0.224	-0.003	-0.005	-0.005	-0.005	-0.005	-0.004	-0.001	-0.014

The CN-cycle neutrino fluxes can be used as a direct probe of the core C and N abundances only to the extent that other SSM uncertainties can be controlled. Uncertainties in S-factors can in principle be improved through better laboratory measurements. In contrast, there may be no effective strategy to reduce the "environmental" uncertainties. These uncertainties often primarily affect the core temperature. For example, when metal abundances are varied, the SSM core temperature responds to the resulting changes in opacity and mean molecular weight: high metallicity cores are hotter. Neutrino fluxes also respond, reflecting their underlying power-law dependences on temperature.

The dependence of the fluxes on environmental and other parameters can be determined from the logarithmic derivatives of Tables I and II,

$$\begin{aligned}\frac{\phi(^{13}\mathrm{N})}{\phi(^{13}\mathrm{N})^{\mathrm{SSM}}} &= \left[L_\odot^{4.535} O^{1.487} A^{0.932} D^{0.337}\right] \\ &\times \left[\mathrm{S}_{11}^{-2.166}\ \mathrm{S}_{33}^{0.031}\ \mathrm{S}_{34}^{-0.062}\ \mathrm{S}_{17}^{0.0}\ \mathrm{S}_{e7}^{0.0}\ \mathrm{S}_{114}^{0.747}\right] \\ \times \Big[x_C^{0.856} x_N^{0.165} x_{\mathrm{O}}^{0.082} &x_{\mathrm{Ne}}^{0.058} x_{\mathrm{Mg}}^{0.049} x_{\mathrm{Si}}^{0.111} x_{\mathrm{S}}^{0.081} x_{\mathrm{Ar}}^{0.021} x_{\mathrm{Fe}}^{0.294}\Big]\end{aligned} \tag{4}$$

where each parameter on the right-hand side represents a $\beta_j/\beta_j^{\mathrm{SSM}}$. The luminosity, opacity, solar age, and the diffusion parameters are denoted by $L_\odot$, O, A, and D, while S and x denote S-factor or abundance ratios. Similarly,

$$\begin{aligned}\frac{\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})^{\mathrm{SSM}}} &= \left[L_\odot^{5.942} O^{2.034} A^{1.364} D^{0.382}\right] \\ &\times \left[\mathrm{S}_{11}^{-2.912}\ \mathrm{S}_{33}^{0.024}\ \mathrm{S}_{34}^{-0.052}\ \mathrm{S}_{17}^{0.0}\ \mathrm{S}_{e7}^{0.0}\ \mathrm{S}_{114}^{1.00}\right] \\ \times \Big[x_C^{0.815} x_N^{0.217} x_{\mathrm{O}}^{0.112} &x_{\mathrm{Ne}}^{0.081} x_{\mathrm{Mg}}^{0.069} x_{\mathrm{Si}}^{0.150} x_{\mathrm{S}}^{0.109} x_{\mathrm{Ar}}^{0.028} x_{\mathrm{Fe}}^{0.397}\Big]\,.\end{aligned} \tag{5}$$

The ^{15}O νs are of more interest experimentally, because their higher energy provides a window for observation in

a scintillation detector, as discussed in [8]. From these expressions and from the "reasonable ranges" for input SSM parameters given in Table III, one can then identify the principal sources of SSM uncertainty in neutrino flux predictions. The ranges assigned in Table III to the metal abundances are of particular concern: the large differences between GS98 and AGSS09 reflect the tension between helioseismology and 3D modeling of the solar atmosphere. For each metal, we assign to its abundance an uncertainty formed by two contributions. On one hand, following [16], a *systematic* component based on the differences between the GS98 and AGSS09 compositions given by

$$\frac{\Delta\beta_i}{\beta_i} = 2\left|\frac{\text{Abundance}_i^{\text{GS98}} - \text{Abundance}_i^{\text{AGSS09}}}{\text{Abundance}_i^{\text{GS98}} + \text{Abundance}_i^{\text{AGSS09}}}\right| \quad (6)$$

to which we add in quadrature the *observational* uncertainty taken as the uncertainty quoted in the latest solar abundance compilation [10]. This is a conservative approach for assigning abundance uncertainties but it is appropriate for the present work, as it leads to robust upper limits to the precision with which solar neutrino experiments can constrain solar interior properties.

It is reasonable to treat the effects of abundances in Eqs. (4) and (5) as an overall scaling of metallicity, as the differences between the GS98 and AGSS09 abundances are effectively systematic in the net metallicity. With this assumption, the dominant SSM uncertainty in Eq. (5) is the core abundance of C + N which, if changed systematically over a range equivalent to the GS98-AGSS09 difference, alters the ^{15}O neutrino flux by 30.7%. This is the sensitivity we want to exploit, in using CN neutrinos as a probe of core metallicity. The next largest certainty comes from the 11 environmental parameters, 16.5%: thus the environmental uncertainties are the primary factor inhibiting our use of neutrinos as a probe of composition. The uncertainty coming from the S-factors, 7.7%, is entirely dominated by S_{114}, which alone contributes 7.2%.

Now the nuclear physics uncertainties can be reduced with effort: in [5] possible steps to improve existing measurements of S_{114} are described. But we have less control over the environmental parameters, so an alternative strategy is needed to address these uncertainties. We use the well-measured flux of ^{8}B as a solar thermometer, to remove as much of the environmental dependence as possible.

We form a weighted ratio of the ^{15}O ν and ^{8}B ν fluxes to eliminate the dependence on T_c to the extent possible, and thus to minimize the dependence on 10 of the 11 environmental parameters of Tables I and II: we do not include the diffusion coefficient, as this parameter plays a special role in the relationship between contemporary flux measurements and the primordial abundances we seek to constrain. The CN neutrino fluxes are more sensitive to diffusion, as Table I shows. All neutrino fluxes respond similarly to changes in core temperature induced by gravitational settling. However, the ^{15}O flux has an additional dependence on changes in the ^{12}C and ^{14}N core abundances, as the rate is proportional to those abundances, for constant temperature. Thus the analysis is done in a way that isolates this additional dependence.

To exploit the well-measured flux of ^{8}B neutrinos as a thermometer in this way, one must determine the linear correlations between $\ln(\phi(^{13}\text{N}))$ and $\ln(\phi(^8\text{B}))$ and between $\ln(\phi(^{15}\text{O}))$ and $\ln(\phi(^8\text{B}))$. While this can be done by direct Monte Carlo SSM calculations (see discussion below), it was shown in [8] that such an exercise is largely equivalent to minimizing the dependence on net logarithmic derivatives. The solution to the minimization is a power law of the N observables, $\prod_{i=1}^{N}(\frac{\phi_i}{\phi_i^{\text{SSM}}})^{b_i^k}$, with exponents b_i^k given by the eigenvector with minimum eigenvalue of the nuisance parameters error matrix [17]

$$\mathcal{M}_{il} = \sum_{j=1}^{n}\left(\frac{\Delta\beta_j}{\beta_j}\right)^2 \alpha(i,j)\alpha(l,j)\,. \quad (7)$$

The computation of the matrices $\mathcal{M}_{^8\text{B},^{13}\text{N}}$ and $\mathcal{M}_{^8\text{B},^{15}\text{O}}$ is straightforward. The direction of the smallest eigenvalue is, respectively,

$$\begin{aligned}\frac{\phi(^{13}\text{N})}{\phi(^{13}\text{N})^{\text{SSM}}}\Big/\left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})}\right]^{0.576} &= x_C^{0.840}x_N^{0.161}D^{0.183}\\ &\times\left[L_\odot^{0.553}O^{-0.017}A^{0.157}\right]\\ &\times\left[\text{S}_{11}^{-0.639}\ \text{S}_{33}^{0.264}\ \text{S}_{34}^{-0.526}\ \text{S}_{17}^{-0.576}\ \text{S}_{e7}^{0.576}\ \text{S}_{114}^{0.743}\right]\\ &\times\left[x_{\text{O}}^{0.002}x_{\text{Ne}}^{-0.005}x_{\text{Mg}}^{-0.004}x_{\text{Si}}^{0.0}x_{\text{S}}^{0.0}x_{\text{Ar}}^{0.001}x_{\text{Fe}}^{0.005}\right]\end{aligned} \quad (8)$$

and

$$\begin{aligned}\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})^{\text{SSM}}}\Big/\left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})}\right]^{0.785} &= x_C^{0.794}x_N^{0.212}D^{0.172}\\ &\times\left[L_\odot^{0.515}O^{-0.016}A^{0.308}\right]\\ &\times\left[\text{S}_{11}^{-0.831}\ \text{S}_{33}^{0.342}\ \text{S}_{34}^{-0.685}\ \text{S}_{17}^{-0.785}\ \text{S}_{e7}^{0.785}\ \text{S}_{114}^{0.995}\right]\\ &\times\left[x_{\text{O}}^{0.003}x_{\text{Ne}}^{-0.005}x_{\text{Mg}}^{-0.003}x_{\text{Si}}^{-0.001}x_{\text{S}}^{-0.001}x_{\text{Ar}}^{0.001}x_{\text{Fe}}^{0.003}\right]\end{aligned} \quad (9)$$

The important dependences on opacity and metallicity (other than C and N) have been almost entirely removed. While some residual dependence on luminosity and age remains, these parameters have relatively small uncertainties. Once the ^{8}B neutrino thermometer has removed the environmental effects, we find that the ^{15}O neutrino flux varies linearly under scaling of the ^{12}C and ^{14}N abundances ($0.794 + 0.212 = 1.006 \sim 1$). This dependence can be made more explicit in Eq. (9) by the replacement

$$x_C^{0.794}x_N^{0.212} \Rightarrow \left[\frac{N_{\text{C}}+N_{\text{N}}}{N_{\text{C}}^{\text{SSM}}+N_{\text{N}}^{\text{SSM}}}\right] \quad (10)$$

TABLE III: Estimated 1σ uncertainties in solar (from Bahcall, Serenelli, and Basu [14] and Fiorentini and Ricci [15]) and nuclear physics (from Adelberger *et al.* [5]) uncertainties, and their influence on flux predictions, computed from the partial derivatives of Table I.

β_j	Value	$\frac{\Delta\beta_j}{\beta_j}$(%)	$\frac{\Delta\phi(^8\mathrm{B})}{\phi(^8\mathrm{B})}$ (%)	$\frac{\Delta\phi(^7\mathrm{Be})}{\phi(^7\mathrm{Be})}$(%)	$\frac{\Delta\phi(^{13}\mathrm{N})}{\phi(^{13}\mathrm{N})}$(%)	$\frac{\Delta\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})}$ (%)
$L_\odot$	3.842×10^{33} ergs/s	0.4	2.8	1.4	1.8	2.4
Opacity	1.0	2.5	6.5	3.0	3.7	5.1
Age	4.57 Gyr	0.44	0.59	0.33	0.41	0.60
Diffusion	1.0	15.0	4.0	1.9	5.1	5.7
p+p	$(4.01\pm0.04)\times10^{-25}$ MeV b	1.0	2.6	1.0	2.2	2.9
^{3}He+^{3}He	(5.21 ± 0.27) MeV b	5.2	2.1	2.2	0.16	0.12
^{3}He+^{4}He	(0.56 ± 0.03) MeV b	5.4	4.3	4.6	0.33	0.28
p+^{7}Be	(20.8 ± 1.6) eV b	7.7	7.7	0.0	0.0	0.0
e+^{7}Be		2.0	2.0	0.0	0.0	0.0
p+^{14}N	(1.66 ± 0.12) keV b	7.5	0.05	0.0	5.6	7.5

TABLE IV: Estimated 1σ historical ("conservative") uncertainties in AGSS98 abundances, as defined in Bahcall and Serenelli [16]. The corresponding uncertainties in the neutrino fluxes are computed from the partial derivatives of Table II.

β_j	$\frac{\Delta\beta_j}{\beta_j}$(%)	$\frac{\Delta\phi(^8\mathrm{B})}{\phi(^8\mathrm{B})}$(%)	$\frac{\Delta\phi(^7\mathrm{Be})}{\phi(^7\mathrm{Be})}$(%)	$\frac{\Delta\phi(^{13}\mathrm{N})}{\phi(^{13}\mathrm{N})}$(%)	$\frac{\Delta\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})}$(%)
C	24.6	0.66	0.05	21.1	20.1
N	24.6	0.17	0.02	4.1	5.3
O	35.0	4.9	2.2	2.9	3.9
Ne	45.3	4.9	2.5	2.6	3.7
Mg	11.8	1.1	0.59	0.58	0.81
Si	11.8	2.3	1.2	1.3	1.8
S	13.8	1.9	1.0	1.1	1.5
Ar	34.9	1.2	0.66	0.73	0.98
Fe	11.8	5.9	2.4	3.5	4.7

where N_C and N_N are the number densities of C and N. That is, the ^{15}O neutrino flux depends effectively only on the sum of the number densities. The exponents appearing in Eq. (9) depend weakly on the SSM about which the variations are made. We use the SFII-GS98 SSM where $N_\mathrm{C}^\mathrm{SSM}/N_\mathrm{N}^\mathrm{SSM} \sim 0.80/0.20$; the same ratio occurs in solar models using the solar composition from [10].

For ^{15}O, the case of most interest experimentally, the observable on the left hand side responds linearly to any scaling of the N and C primordial abundances. Diffusion, using Table III, creates a 2.6% uncertainty in relating contemporary flux measurements to the primordial abundance. This 2.6% is virtually all that remains of the original 16.5% SSM environmental uncertainty of Eq. (5): the ^{8}B neutrino thermometer has reduced the uncertainties associated with the remaining 10 parameters to below 0.35%. The third term on the right, contributions from the S-factors, is now the dominant theoretical uncertainty in the relationship between primordial C+N and neutrino flux measurements, contributing 10.6% to the error budget.

The explicit treatment of diffusion, effectively grouping diffusion with the C+N abundance, differs from the original work of [8]. This choice is made for simple physical reasons, that neutrino flux measurements respond to contemporary core abundances, yet the parameters needed in the SSM, which describes the Sun's evolution from the onset of nuclear burning, are primordial. Thus the relationship we establish between primordial core C+N and contemporary CN neutrino fluxes has a dependence on diffusion that should be made explicit, as we have done here. Indeed, the effects of diffusion are not inconsequential: the SFII-GS98 and SFII-AGSS09 SSM metal profiles of Fig. 1 show that diffusion over 4.6 Gyr of solar evolution leads to nontrivial structures. Fortunately for our present goals, helioseismology is sensitive to He and metal diffusion: the 15% uncertainty on the diffusion coefficient (see Table III) is a credible limit on diffusion uncertainties because of helioseismic constraints.

Once this dependence on diffusion is separated out, it becomes apparent that almost all of the residual SSM "environmental" dependence identified in [8] – variations in 11 SSM parameters producing a net uncertainty of 2.6% – is due to the diffusion coefficient. The correlations illustrated in Fig. 3 of Ref. [8] are redone in Fig. 2, with the removal of this one parameter. The net uncer-

tainty due to uncorrelated variations in the remaining 10 parameters is now reduced to 0.3%, as mentioned above.

The Super-Kamiokande measurement of the ^{8}B flux has reached a precision of 2%. Borexino has set the strongest constraint on the CNO solar neutrino interaction rate (<7.9 counts/(day x 100 ton) at 95 % C.L.) and their latest purification campaign has resulted in a much lower background level, what opens the possibility of the first detection of CNO neutrinos [18]. SNO+ has the potential to measure the ^{15}O flux to an accuracy of about 10% in three years of running, if the detector design goals are reached [19]. Thus the current theoretical $\sim$ 30% uncertainty in the core C+N abundance could be substantially reduced by a neutrino measurement. In fact, the limiting uncertainty appears to be the nuclear physics, specifically S_{17} (7.7%) and S_{114} (7.5%). Both of these reactions were recently evaluated by the nuclear astrophysics community [5]. The uncertainty in S_{17} is dominated by the theory used to fit and extrapolate measurements – the experimental contribution to the S_{17} error is 3.4% [5]. As *ab initio* methods may soon be available for such systems [20], the situation could improve substantially. In the case of S_{114} a program of needed work was outlined in [5], including new measurements to constrain the transitions to the 6.79 and 6.17 MeV states in ^{15}O. We conclude that it should be possible to significantly reduce the overall uncertainty from the nuclear physics.

The expressions above are valid for the neutrino fluxes at the source. We need to account for the effects of neutrino flavor conversion, as this alters the ratio of detected ^{15}O to ^{8}B neutrinos in detectors based on ν_x-e scattering. ^{8}B neutrino oscillation probabilities are smaller because their energies correspond to the matter dominated flavor conversion while the CN neutrinos are in the vacuum oscillation regime with small matter effects. We use the most up-to-date neutrino oscillation analysis [21] to estimate the uncertainty due to the neutrino parameters, with lowers the weak-interactions uncertainty in our analysis to $\pm$ 3%.

The analysis suggests that a neutrino determination of the C + N content of the core at a confidence level of $\sim$ 10% is quite feasible with the future measurements. This assumes a 7% ^{15}O neutrino measurement and modest progress in lowering nuclear physics uncertainties to the same level. This should be compared to the current metallicity controversy, $\sim$ 30%. Such a measurement would also constitute the first direct experimental test of an important SSM assumption, that the primordial core and modern solar atmosphere metallicities are the same, once corrections are made for the effects of diffusion. Other sources of uncertainty – the Super-Kamiokande measurement of the ^{8}B neutrino rate for elastic scattering (ES), the SNO combined analysis constraining weak interaction parameters, and the influence of diffusion (the one effect intrinsic to the SSM that can not be adequately subtracted using the ^{8}B neutrino thermometer) – are all of minor importance, contributing to the error budget at $\sim$ 3%.

There is a second constraint on core composition that could be obtained with CN neutrinos and that is inherently interesting because the observable is exceptionally free of SSM uncertainties. This constraint, however, requires a measurement of the ^{13}N neutrinos. If the pep shoulder is seen and if the level of background in the ES measurements can be kept low (or reliably subtracted), the remaining counts in the $\sim$ 1 MeV region could be associated with the ^{13}N and ^{15}O neutrinos. One important observation is that the relative contributions of the two neutrino sources to the total rate vary with the electron recoil energy, due to the different neutrino energy distributions. This could allow the experimenters to separate the two CN-neutrino flux components. In Table V, we show the relative contribution to the scattering rate as a function of the measured recoil electron energy interval. We have used the energy resolution of the Borexino detector and the ^{13}N and ^{15}O energy distribution of neutrinos given by the SFII-GS98 model. In the interesting energy range between the ^{7}Be and pep shoulders, the relative contribution to the rate varies by one order of magnitude. The higher energy bins in this range are strongly dominated by the ^{15}O neutrino flux contribution and the error estimate of the flux will be comparable to the experimental error in this energy region. The lower energy bins have a significant contribution from the ^{13}N neutrino flux. Thus this flux component could also be determined from the data, though with a larger uncertainty due to the need to separate this component from the dominant ^{15}O contribution. An important caveat is the assumption there are no unidentified background sources that might mimic the ^{13}N neutrino signal.

As noted previously, the CN cycle has not reached equilibrium in the Sun apart from its central core. The lifetime of ^{14}N, determined by the rate of ^{14}N(p,γ), is less than the solar age only for $T_7 \gtrsim 1.33$ (T_7 is the temperature in 10^7K). But at this temperature the lifetime of ^{12}C is $\sim 2 \cdot 10^7$ years. Thus somewhat outside the central core, say at $T_7 \sim 1.15$, there will be very little ^{14}N burning, and also very little ^{12}C burning, as the primordial carbon would have been consumed long ago. Still further outward, where $T \sim 10^7$K, the ^{12}C lifetime is comparable to the solar age. This is the region in the contemporary Sun where primordial ^{12}C is being burned. We conclude that CN neutrinos are coming from two distinct regions. The CN cycle is in equilibrium deep in the core, producing approximately equal numbers of ^{15}O and ^{13}N neutrinos, while well away from this region, in the cooler outer core at $T \sim 10^7$K, primordial ^{12}C is burning to ^{14}N, producing only low-energy ^{13}N neutrinos.

That is, the unequal fluxes of ^{13}N and ^{15}O neutrinos are a reflection of the burning of primordial ^{12}C in the outer core. We can test for this effect by comparing these

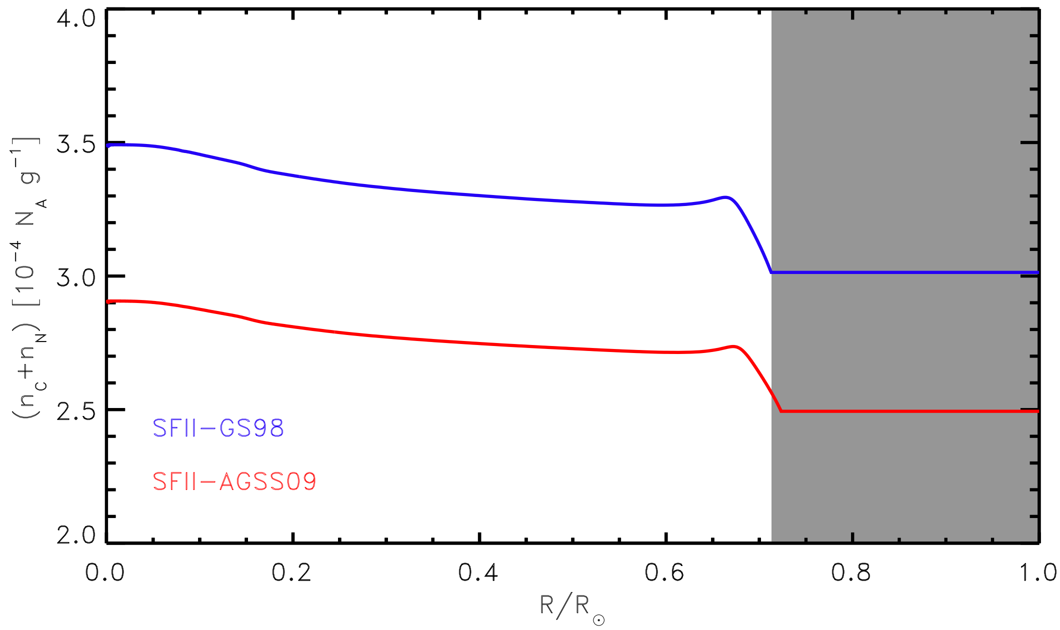

FIG. 1: The modern Sun's carbon plus nitrogen number profiles in the SFII-GS98 and SFII-AGSS09 SSMs, showing the effects of diffusion over 4.6 Gyr of stellar evolution. The shaded area denotes the convective envelope.

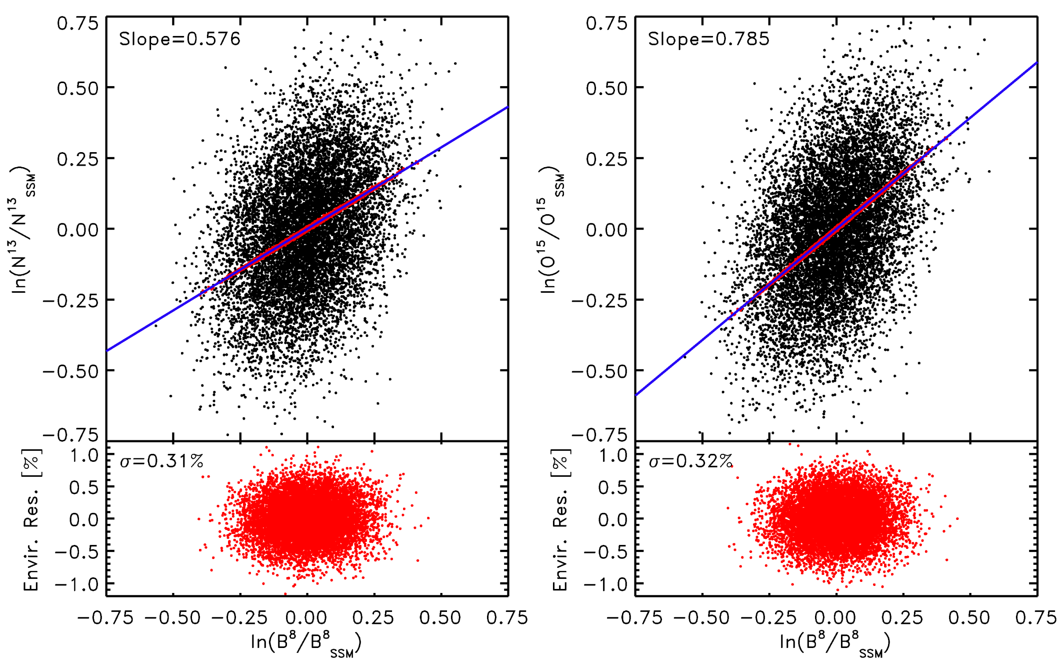

FIG. 2: Solar neutrino fluxes from models in which all the parameters (black) or 10 of the 11 environmental parameters (red) – the diffusion coefficient is held fixed – are varied. At this resolution red points are indistinguishable from a line. The two upper panels show the correlation between the ^{8}B flux and the two CN-cycle neutrino fluxes. The slopes of the correlations between fluxes when only 10 environmental parameters are varied are given in the plots. The residuals from the fits, 0.31% and 0.32%, are shown in the lower panels. The results can be compared to those of Fig. 3 in [8], where residuals of 2.8% and 2.6% were obtained for the ^{13}N and ^{15}O fluxes, respectively, when diffusion was included as an 11th environmental parameter.

TABLE V: Ratio of the scattering rate by ^{15}O and ^{13}N neutrino with electrons in a Borexino-like detector.

E (MeV)	[0.70,0.75]	[0.75,0.80]	[0.80,0.85]	[0.85,0.90]	[0.90,0.95]	[0.95,1.0]
$R(^{15}\mathrm{O})/R(^{13}\mathrm{N})$	2.1	2.6	3.6	5.5	10.2	24.7

fluxes, treating the ^{15}O neutrino flux as the thermometer. In the exercise to find the linear correlation between the logarithmic fluxes, we now include diffusion among the environmental parameters, as it should affect ^{13}N and ^{15}O neutrino rates almost equally. We find

$$\frac{\phi(^{13}\mathrm{N})}{\phi(^{13}\mathrm{N})^{\mathrm{SSM}}} \Big/ \left[\frac{\phi(^{15}\mathrm{O})}{\phi^{\mathrm{SSM}}(^{15}\mathrm{O})}\right]^{0.776} = x_C^{0.224} x_N^{-0.003} \mathrm{S}_{112}^{-0.008}$$
$$\times \left[L_\odot^{-0.075} O^{-0.091} A^{-0.126} D^{-0.041}\right]$$
$$\times \left[\mathrm{S}_{11}^{0.093}\ \mathrm{S}_{33}^{0.012}\ \mathrm{S}_{34}^{-0.022}\ \mathrm{S}_{17}^{0.0}\ \mathrm{S}_{e7}^{0.0}\ \mathrm{S}_{114}^{-0.029}\right]$$
$$\times \left[x_{\mathrm{O}}^{-0.005} x_{\mathrm{Ne}}^{-0.005} x_{\mathrm{Mg}}^{-0.005} x_{\mathrm{Si}}^{-0.005} x_{\mathrm{S}}^{-0.004} x_{\mathrm{Ar}}^{-0.001} x_{\mathrm{Fe}}^{-0.014}\right]. \tag{11}$$

The residual environmental uncertainty is only $\sim 0.7\%$. For the nuclear part we have made explicit the very small dependence on the S-factor of the ^{12}C(p,γ)^{13}N reaction rate, S_{112}. The total nuclear uncertainty in the above expression is only 0.3%. We note here the dependence of this ratio on the nitrogen abundance is partially accidental. The exponent that minimizes the environmental uncertainties, 0.776, is very close to the ratio of the partial derivatives of these fluxes with respect to the N abundance 0.165/0.217=0.760 (see Table II). However, we find that even for unrealistically large variations of more than a factor of 2 in the assumed composition of the Sun, this ratio varies little, between 0.71 and 0.81. Therefore, the cancellation of the nitrogen abundance in Eq. 11 will always occur at a level better than 0.5%. A similar conclusion can be drawn with respect to the S_{114} astrophysical factor, which is also accidentally cancelled for the same reason.

Other constraints: While we have focused on metallicity and the CN-cycle neutrinos, due to the troubling solar abundance problem, the use of SSM power-law temperature dependences to extract parameter constraints is a general strategy for exploiting the Sun as a laboratory. For example, the primordial ^{4}He abundance [22] was recently constrained using very similar arguments. Another example we discuss here is the possibility of using the SSM to cross-check laboratory measurements of S-factors. S_{17} is an important example because of the relatively large uncertainty in this S-factor and because of its importance to the branching between the ppII and ppIII cycles. The analysis is quite simple, a comparison the the ^{7}Be and ^{8}B fluxes, neither of which has any anomalous dependence on metal abundances or diffusion. Thus we can optimize over all 13 non-nuclear parameters, with the anticipation that the residuals will be small in each of these. Following the previous calculation, we find

$$\frac{\phi(^7\mathrm{Be})}{\phi(^7\mathrm{Be})^{\mathrm{SSM}}} \Big/ \left[\frac{\phi(^8\mathrm{B})}{\phi^{\mathrm{SSM}}(^8\mathrm{B})}\right]^{0.465} =$$
$$\left[L_\odot^{0.219} O^{-0.004} A^{0.135} D^{0.002}\right] \times$$
$$\left[\mathrm{S}_{11}^{0.209}\ \mathrm{S}_{33}^{-0.240}\ \mathrm{S}_{34}^{0.479}\ \mathrm{S}_{17}^{-0.465}\ \mathrm{S}_{e7}^{0.465}\ \mathrm{S}_{114}^{-0.004}\right] \times$$
$$\left[x_{\mathrm{C}}^{-0.011} x_{\mathrm{N}}^{-0.002} x_{\mathrm{O}}^{-0.003} x_{\mathrm{Ne}}^{0.004} x_{\mathrm{Mg}}^{0.007} x_{\mathrm{Si}}^{0.015} x_{\mathrm{S}}^{0.011} x_{\mathrm{Ar}}^{0.003} x_{\mathrm{Fe}}^{-0.026}\right]$$
$$\sim \left[\frac{\mathrm{S}_{11}}{\mathrm{S}_{33}}\right]^{0.24} \left[\frac{\mathrm{S}_{34}\mathrm{S}_{e7}}{\mathrm{S}_{17}}\right]^{0.48} F_{\mathrm{SSM}}^{\mathrm{nonnuclear}} \tag{12}$$

The error introduced by grouping the astrophysical factors in the last expression is only 0.1%. In this expression the factor $F_{\mathrm{SSM}}^{\mathrm{nonnuclear}}$ represents the contributions from the 13 non-nuclear uncertainties in the SSM: using the exponents above and the β_j of Tables III and IV, one finds that this contribution deviates from unity by $\sim \pm 0.5\%$, and therefore plays no significant role. Thus effectively we have a direct relationship between neutrino flux measurements and nuclear cross sections. The left-hand side of Eq. (11) is the product of two factors. The first, $[\mathrm{S}_{11}/\mathrm{S}_{33}]^{0.24}$, is uncertain to 1.3%, using the evaluations of Solar Fusion II, with the error dominated by that in S_{33}. The second, $[\mathrm{S}_{34}\mathrm{S}_{e7}/\mathrm{S}_{17}]^{0.48}$, is uncertain to 4.6%, treating all uncertainties as uncorrelated. Thus the left-hand side of Eq. (11) is 1 ± 0.048, when all uncertainties are combined in quadrature.

The right-hand side can be evaluated from the results of global solar neutrino flux analyses that incorporate the neutrino oscillation results important to mixing angle determinations (as the fluxes are the unoscillated instantaneous ones)[4]. The analysis is done in terms of the normalizations provided by SFII-GS98 SSM best values, for consistency with the logarithmic derivatives we employ: $\phi(^7\mathrm{Be}) = 5.00 \times 10^9/\mathrm{cm}^2\mathrm{s}$ and $\phi(^8\mathrm{B}) = 5.58 \times 10^6/\mathrm{cm}^2\mathrm{s}$. The experimental fluxes are $4.82(1 \pm 0.045) \times 10^9/\mathrm{cm}^2\mathrm{s}$ and $5.00(1 \pm 0.03) \times 10^6/\mathrm{cm}^2\mathrm{s}$. Consequently the left-hand side is $1.016(1 \pm 0.047)$. If the SFII-AGSS09 SSM best values are used to normalize the left hand side, the result is virtually unchanged, $1.015(1 \pm 0.047)$.

This result is significant: the constraint imposed on the ratio of S-factors $[\mathrm{S}_{11}/\mathrm{S}_{33}]^{0.24}\,[\mathrm{S}_{34}\mathrm{S}_{e7}/\mathrm{S}_{17}]^{0.48}$ relative to SFII best values, using all information available from laboratory astrophysics, has the same precision as the similar ratio we can deduce from neutrino flux measurements, if we employ the SSM to predict the dependence of the fluxes on input S-factors, and if we constrain all non-nuclear parameters to vary only within the

ranges allowed by their currently assigned uncertainties. This result was achieved by identifying a specific ratio of ^{7}Be and ^{8}B fluxes that the SSM predicts will exhibit the minimum uncertainty to variations in the 13 nonnuclear parameters. The two independent constraints – the left- and right-hand sides of Eq. (11) – are in excellent agreement, a result that reflects the concordance between neutrino flux observations and laboratory nuclear cross section measurements, in the context of the SSM. As the uncertainty of the neutrino-flux result for this S-factor ratio is dominated almost entirely by that for the ^{7}Be neutrino flux, further improvements in the Borexino result (or new results from a next-generation experiment such as SNO+) would make the neutrino-flux S-factor constraint the more precise one.

While this test of concordance between neutrino flux measurements and laboratory measurements of S-factors is our main point, one can be more aggressive and ask whether new S-factor information can be derived directly from neutrino flux measurements. As the most uncertain of the S-factors is S_{17}, what level of precision is needed in neutrino flux measurements to improve our knowledge of this cross section? Equation (11) can be rewritten

$$\begin{aligned} S_{17} \;&=\; S_{34} S_{e7} \left[\frac{S_{11}}{S_{33}}\right]^{0.5} \left[F_{\rm SSM}^{\rm nonnuclear}\right]^{2.08} \\ &\times\; \frac{\phi(^8\mathrm{B})}{\phi^{\rm SSM}(^8\mathrm{B})} \Big/ \left[\frac{\phi(^7\mathrm{Be})}{\phi(^7\mathrm{Be})^{\rm SSM}}\right]^{2.08} \end{aligned} \tag{13}$$

Note that the simple exponent 0.5 on the S-factor ratio $[S_{11}/S_{33}]$ is not accidental, but reflects the fact that the ^{3}He abundance has achieved equilibrium in the region of the core where ^{7}Be and ^{8}B neutrinos are being produced. The number densities for ^{3}He and protons are then related by

$$\left[\frac{N_3}{N_p}\right]_{\rm equil} = \sqrt{\frac{\lambda_{pp}}{2\lambda_{33}}} \tag{14}$$

where λ_{pp} and λ_{33} are the local rates proportional to the respective S-factors. Effectively Eq. (13) states that laboratory uncertainties in S_{17} (currently 7.5%) can be traded off against those in S_{34} (5.4%) and $\phi(^7\mathrm{Be})$, the most poorly known quantities on the right-hand side. (Remember that all S-factors are normalized to their SFII best values.) Adding errors in quadrature, we find that this alternative determination yields $S_{17} = 0.967(1\pm0.117)$. The result will not be competitive, given the current laboratory precision of 7.5%, unless the uncertainties on both S_{34} and $\phi(^7\mathrm{Be})$ are reduced to $\sim$3%.

Summary: We have refined the previous arguments of [8] to show that future ^{15}O neutrino flux measurements have the potential to constrain the primordial core metallicity of C+N to an accuracy of $\sim$ 10%. This would be a very significant result, given that differences in recent abundance determinations exceed 30%. The method exploits the additional linear dependence on metallicity of CN cycle burning, and is limited primarily by expected uncertainties of future experiments like SNO+ and by current uncertainties in laboratory measurements of nuclear cross sections. The nonnuclear uncertainties in the relationship we derived, previously determined in Monte Carlo studies to be less than $\sim$ 3% [8], are in fact negligible apart from one parameter, the diffusion coefficient. The dependence on diffusion is natural, reflecting the fact that contemporary neutrino flux measurements are being used to constrain primordial abundances, not present-day core abundances. Our primary test of diffusion and its uncertainties comes from helioseismology, which provided the initial motivation for including He and heavy-element diffusion in solar models. We also point out the possibility – speculative experimentally, but intriguing theoretically – that by also measuring the ^{13}N solar neutrinos, one could determine the separate core abundances of C and N. The present-day burning of primordial C in the cooler outer core of the Sun contributes to the ^{13}N solar neutrino flux.

The idea behind the metallicity extraction is a general one: forming ratios of neutrino fluxes that minimize the sensitivity to core temperature and thus to solar model uncertainties. We developed a second example of such a minimum-uncertainty SSM ratio – a comparison of ^{7}Be and scaled ^{8}B neutrino fluxes – that isolates a specific ratio of S-factors. We demonstrated that the precision to which this ratio is known from direct laboratory measurements is in fact identical to the precision it can be determined from measured solar neutrino fluxes and the SSM, given existing uncertainties on nonnuclear input parameters to that model. Thus neutrino flux measurements have now reached the precision where meaningful consistency tests with laboratory cross sections can be done. In the example we developed, the laboratory cross-section measurements and neutrino fluxes were found to be in excellent agreement.

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